Screening as a Unified Theory of Delinquency, Renegotiation, and Bankruptcy

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Motivation

- (Stages of) Default in consumer credit
  - Delinquency: payments are overdue by at least 60 days
  - Some, but not all, delinquent borrowers end up in bankruptcy
  - Lenders sometimes renegotiate with delinquent borrowers to prevent bankruptcy and achieve debt settlement
- There is no (simple) theory that models all these stages
  - More on related literature later
What We Do

• Construct a very simple model where delinquency, renegotiation, and bankruptcy all occur in equilibrium

• Key model ingredient: adverse selection
  ○ A borrower’s bankruptcy cost is her private information
    - Lenders often do not observe personal characteristics that affect a borrower’s willingness to pay

• All three phenomena are generated by a simple screening mechanism

• They match the default stages that we think of in reality
  ○ Some borrowers choose not to repay → become delinquent
  ○ Lenders renegotiate with some delinquent borrowers → debt settlement
  ○ In absence of renegotiation, delinquency leads to bankruptcy
What Others Do

- Consumer debt literature
  - Focuses on bankruptcy, but largely abstracts from delinquency, and especially renegotiation

- Sovereign debt literature
  - Focuses on default and (sometimes) renegotiation
  - Seldom distinguishes between ‘delinquency’ and ‘bankruptcy’ (∼ ‘autarky’); default usually means one of the two

- In terms of the modeling approach
  - Our paper is related to the literature on optimal mechanisms of selling a good to heterogeneous risk-averse buyers
What We Do (Continued)

• Comparative Statics
  ◦ Reasonable predictions about how the bankruptcy rate varies with debt and income

• Application: Government intervention in debt restructuring
  ◦ Example: Mortgage Modification Program
**Environment**

- One lender, one borrower, one period

- **Borrower**
  - Risk averse, has utility function $u(c)$, $u' > 0$, $u'' < 0$
  - Has income $I$
  - Owes debt to the lender
    - For simplicity, we abstract from where debt comes from
  - Has the option of declaring bankruptcy
    - Idiosyncratic cost of bankruptcy, $\theta \in \{\theta_L, \theta_H\}$, unobservable to the lender, $\Pr\{\theta = \theta_H\} = \gamma$
    - Bankruptcy yields $v(I, \theta)$ to the borrower, zero to the lender
    - $v(I, \theta_L) > v(I, \theta_H)$ for any $I$

- **Lender**
  - Risk neutral
  - Demands repayment
Contracts

• Designed by the lender

• Deterministic contract: repayment $R$
  ○ A borrower of type $i$ accepts if and only if $u(I - R) \geq v(I, \theta_i)$

• Two possible equilibria with deterministic contracts:
  ○ Offer $R_L$: $u(I - R_L) = v(I, \theta_L) \Rightarrow$ attract both types (pooling)
  ○ Offer $R_H$: $u(I - R_H) = v(I, \theta_H) \Rightarrow$ attract only high type (exclusion)
  ○ Which contract yields higher profits to the lender depends on $\gamma$

• The lender can potentially do better by offering a pair of random contracts (screening)
Screening Contract

Pair of contracts: \( R_1, (R_2, p) \)

- Deterministic contract (for the high type): \( R_1 \)
- Random contract (for the low type): \( R_2 < R_1 \) with probability \( p \), bankruptcy with probability \( 1 - p \)

- To maximize the lender’s profits:
  - \( R_2 = R_L \) and \( R_1 = R_S < R_H \), where (given \( p \)) \( R_S \) solves
    \[
    u(I - R_S) = p u(I - R_L) + (1 - p) u(I - R_H) = v(I, \theta_L) + (1 - p) v(I, \theta_H)
    \]
  - Low type is indifferent b/w accepting \((R_L, p)\) and bankruptcy
  - High type is indifferent b/w accepting \( R_S \) and \((R_L, p)\)
  - Note: \( p < 1 \) only to keep the high type from accepting the contract meant for the low type
Interpretation of a Screening Contract

The lender

- Offers initial repayment
  - High cost borrowers accept it, low cost borrowers do not — consider these borrowers *delinquent*
- **Renegotiates** with delinquent borrowers — offers a lower repayment — but only with some probability
  - The fraction of borrowers with whom the lender does not renegotiate declare **bankruptcy**
  - The others reach debt settlement
The Lender’s Problem

\[ \max_{p \in [0,1]} \pi(p) \equiv \gamma R_S(p) + (1 - \gamma)pR_L, \]

where \( R_S(p) \) solves

\[ u(I - R_S) = pu(I - R_L) + (1 - p)u(I - R_H) \]

• Note: \( p = 1 \) (\( p = 0 \)) corresponds to pooling (exclusion)

• Denote \( p^* = \arg \max_p \pi(p) \)
Equilibrium Contract

Claim 1

1. If the borrower is risk neutral, then \( p^* \in \{0, 1\} \), i.e., screening is always dominated by either pooling or exclusion

2. If the borrower is risk averse, then \( p^* \in (0, 1) \) for some parameter values
   - In particular, if the lender is indifferent between pooling and exclusion, then the equilibrium contract is a screening one
Introduce Debt Level:

- A borrower owes debt $D$ to the incumbent lender
  - The lender cannot ask for a repayment in excess of $D$
- Previously analyzed “debt overhang” case whenever $D > R^*_S$
- The lender’s problem is now
  \[
  \max_{p \in [0,1], R^D_S} \gamma R^D_S + (1 - \gamma)p R^D_L,
  \]
  subject to
  \[
  u(I - R^D_S) \geq pu(I - R^D_L) + (1 - p)u(I - R_H)
  \]
  and
  \[
  R^D_S \leq D
  \]
  where $R^D_L = \min\{R_L, D\}$
Optimal Contract in the General Case

Proposition

(i) If $D \geq R^*_S$, then there is debt overhang and the lender offers $(R^*_S, (R_L, p^*))$ that solves the unconstrained problem.

(ii) If $D \leq R_L$, then the lender demands repayment $D$, and all borrowers fully repay their debt.

(iii) If $D \in (R_L, R^*_S)$, then the lender performs screening: offers $R^D_S = D$ to the high-cost borrowers and $R_L$ with probability $p^*_D > p^*$ to the low-cost borrowers.
Equilibrium Contracts Under Competition

Repayment demanded from high-cost borrowers, $R_s^C$

Constrained screening

Unconstrained exclusion

Constrained screening

Unconstrained screening

Constrained pooling

Unconstrained pooling

$R_H$

$R_S^*$

$R_L$

Debt, $D$

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Equilibrium Under Competition

Bankruptcy rate,

\[(1 - \gamma)(1 - p^*_C)\]

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Bankruptcy Rate: Comparative Statics

- Bankruptcy rate $\xi$ is increasing in debt, $D \checkmark$

- Comparative statics of $\xi$ with respect to $I$
  - Example: $u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma}$, $v(I, \theta) = u((1 - \theta)I)$

- Within monopolistic screening, $\xi$ is constant in $I$

- But debt threshold for monopoly is increasing in $I$
  - Competition is more likely to be relevant for higher $I$, and the bankruptcy rate is lower with competition $\checkmark$

\[
\begin{align*}
\xi(D, I) &\equiv (1 - \gamma)(1 - p_c^*(D, I)) \\
(1 - \gamma)(1 - p^*) &
\end{align*}
\]
Government Intervention in Mortgage Market

• Modeling private sector debt restructuring is crucial for understanding the effects of government intervention

• Example: Mortgage Modification Program
  ◦ HAMP (Home Affordable Mortgage Program) in 2009
  ◦ Aimed at lowering the foreclosure rate (and the deadweight loss associated with it)

• We will analyze effects of such a program through the lens of our model
  ◦ Intervention may have unintended consequences if its design is naive and ignores the effect on private restructuring
Government Intervention in the Model

- Government intervention in our model:
  - Government steps in if bankruptcy (foreclosure) is initiated
  - Offers repayment $R_G$ with probability $p_G$
  - If accepted, the repayment is transferred to the lender

- Suppose the laissez-faire outcome is unconstrained screening

- Key insights:
  1. The policy can be effective,
     even when government appears to be inactive
  2. The policy can have the opposite effect from the one intended
     — lead to more foreclosures in equilibrium

Note: In our model, intervention is never Pareto improving, since equilibrium is constrained Pareto efficient (the government is subject to the same frictions)
Deterministic Government Intervention \((p_G = 1)\)

- If \(R_G \geq R_H\), the intervention is irrelevant
  - Outcomes same as in \textit{laissez-faire} benchmark
- If \(R_G \leq R_L\), the intervention is completely successful
  - Intervention is similar to lowering debt level below \(R_L\)
  - induces “constrained pooling”: the lender demands \(R_G\), everyone repays (no delinquencies, no foreclosures)
- If \(R_G \in (R_L, R_H)\), the intervention
  - may be completely successful while appearing irrelevant
    - \(R_G\) slightly greater \(R_L\) induce pooling
    - lender demands \(R_L < R_G\), no foreclosures
  - or may “backfire” — increase foreclosure rate
    - when \(R_H\) is close to \(I\), small probability of bankruptcy is enough to induce high-cost borrowers to pay
    - intervention is akin to lowering \(R_H\)
Government Intervention: Numerical Example

![Graph showing bankruptcy rate with and without government intervention]

- **Without intervention**
- **With intervention**

**Bankruptcy rate**

- $R_L$ and $R_H$ represent the lower and higher thresholds, respectively.

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Random Intervention: Additional Insights

1. The intervention can be ineffective although the government is busy preventing foreclosures
   - Consider $R_G = R_L$ and $p_G \leq p^*$
   - The lender adjusts $p$ to offset the intervention
   - The resulting foreclosure rate is same as laissez-faire

2. The program can backfire although the government’s offer is accepted when offered
   - Consider $R_G < R_L$ and $p_G \leq p^*$
   - Affects the lender’s ability to extract repayment not just from the high type, but also from the low type
   - As screening (renegotiation) becomes more costly, the lender may decrease $p$ so much that
   - the resulting foreclosure rate increases instead of decreasing
Government Intervention: Numerical Example

- Bankruptcy rate without intervention
- Bankruptcy rate with intervention
Conclusions

- We constructed a simple model with adverse selection
- Delinquency, renegotiation, and bankruptcy all occur in equilibrium as a result of a simple screening mechanism
- Our model generates reasonable comparative statics with respect to debt and income
- Explicitly modeling private debt restructuring is crucial for analyzing the effects of government intervention
Government Intervention: Numerical Example

**Part a**

- Graph showing the bankruptcy rate $R_B$ against $R_G$.
- Two curves: 
  - Dashed line: Without intervention
  - Solid line: With intervention
- For $R_G = 0$, $R_B$ is 0.04 for both.
- As $R_G$ increases, $R_B$ increases for both.
- The dashed line is consistently below the solid line.
- The slopes suggest a decreasing rate of change for the dashed line compared to the solid line.

**Part b**

- Graph showing the bankruptcy rate $R_B$ against $R_G$.
- Two curves: 
  - Dashed line: Without intervention
  - Solid line: With intervention
- For $R_G = 0$, $R_B$ is 0.36 for both.
- As $R_G$ increases, $R_B$ decreases for both.
- The dashed line is consistently below the solid line.
- The slopes suggest an increasing rate of change for the dashed line compared to the solid line.