Risking Other People’s Money: Gambling, Limited Liability, and Optimal Incentives

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Motivation

• Financial meltdown 2008
  • Ex ante unlikely outcome
  • Ex post AIG, Lehman, Citi, Merrill Lynch, etc. suffered high losses
  • Losses were caused by divisions trading highly risky securities
  • Investors were unable to either monitor or understand actions taken by managers

• Managers enjoy limited liability and their compensation is performance based
Moral Hazard and Optimal Contracting

- Managers may seek private gain by taking on *tail risk*
  - Earn bonuses based on short-term gains
  - Put firm at risk of rare disasters
  - Limited liability leaves them insufficiently exposed to downside risk
  - Is this the result of inefficient contracting?

- Standard contracting models
  - Focus on effort provision
  - Static and dynamic models
  - Rewards for high cash flows can be optimal
  - But does this contract lead to excessive risk-taking?
One-Period Model

• Principal/Investor(s)
  • Risk-neutral
  • Owns the company
  • Value of the company without project is $A$ (large)

• One period risky project with payoff:

\[
Y(q) = \begin{cases} 
1, & \text{with probability } \mu + q\rho \\
0, & \text{with probability } 1 - \mu - q(\rho + \delta). \\
-D, & \text{with probability } q\delta 
\end{cases}
\]

• Project risk
  • Low risk $q = 0$
  • High risk $q = 1$
  • High risk is suboptimal: $\rho - \delta D < 0$
One-Period Model

- Principal hires agent/manager to run the project

- New output $Y$, subject to two-dimensional agency problem:
  - Divert output / shirk for private benefit ($\lambda$)
  - Gamble ($\rho < \delta D$)

- How does the possibility of gambling affect contracting?
One-Period Model

- Contract specifies payoffs \((w_0, w_1, w_d)\)
  - \(w_d = 0\)
  - \(w_1 \geq w_0 + \lambda\)

- No Gambling:
  - \(\rho (w_1 - w_0) \leq \delta w_0 \iff w_0 \geq \rho \lambda / \delta\)
  - Agent must receive sufficient rents to prevent gambling
    - Exp. payoff \(= w_0 + \mu \lambda \geq \rho \lambda / \delta + \mu \lambda = \lambda (\mu + \rho / \delta) \equiv w_s\)

- Gambling:
  - Reduce agent rents: \(w_0 \geq 0\)
    - Exp. payoff \(= w_0 + (\mu + \rho) \lambda \geq \lambda (\mu + \rho) \equiv w_g < w_s\)
  - Suffer expected loss: \(\delta D - \rho \equiv \Delta\)
One-Period Model

- Low risk is more profitable to principal than high risk if
  \[ \mu - w^s \geq \mu - \Delta - w^g \]
  \[ \Delta \geq \lambda \left( \frac{\rho}{\delta} - \rho \right) \]
  - For small \( \delta \) principal would prefer to implement high risk project or not to undertake any project

- Gambling is more costly to prevent when probability of disaster is low
  - Limited liability prohibits harsh punishment of agent for gambling,
  - Expected loss \( \delta w_0 \) is low when \( \delta \) is low,
  - Unless agent’s compensation \( w_0 \) and \( w^s \) are high
Contract Conditional on Disaster

• If we cannot punish agent for gambling it may be cheaper to reward him for not gambling ex post

• Can the agent be rewarded for not gambling ex post?
  • Oil spills
    • Absence does not mean gambling did not occur – perhaps we just got lucky?
  • Earthquakes
    • If the building survives an earthquake, that is evidence that the builder did not cut corners
  • Financial crisis
    • If a bank survives it while other banks fail, that is evidence that the bank did not gamble
Bonus for not Gambling

- No gambling: pay bonus $b$ if no loss ($-D$) given disaster
  \[ \rho (w_1 - w_0) \leq \delta (w_0 + b) \]

- Contract without gambling that maximizes principal payoff:
  \[ w_d = 0, \ w_0 = 0, \ w_1 = \lambda, \ b = \lambda \frac{\rho}{\delta}. \]

- Bonus $b$ may be large, but expected bonus payment is not
  \[ \delta b = \lambda \rho \]

- Exp. payoff for Agent $= \lambda \mu + \delta b = \lambda \mu + \rho \lambda \equiv w^g$

- In that case, no gambling is always optimal
Implementation Using Put Options

• Agent is given out-of-money put options on companies that are likely to be ruined in the "disaster" state
  • Caveat: Agent can collect the payoff from the options only if his company remains in a good shape

• Potential downside of using put options
  • Creates incentives to take down competitors

• Comprehensive cost-benefit analysis is needed
Dynamic Model

- A simple model (DS 2006)
  - Cumulative cash flow: \( dY = \mu \, dt + \sigma \, dZ \)
  - Agent can divert cash flows and consume fraction \( \lambda \in (0, 1] \)
  - Alternative interpretation: drift \( \mu \) depends on agent’s effort
    - Earn private benefits at rate \( \lambda \) per unit reduction in drift

- Gambling with tail risk
  - Gambling raises drift to \( \mu + \rho \): \( dY = (\mu + \rho) \, dt + \sigma \, dZ \)
  - Disaster arrives at rate \( \delta \), destroying the franchise and existing assets \( D \) if the agent gambled
Basic Agency Problem

- Interpretations
  - Cash Flow Diversion
  - Costly Effort (work/shirk)

Diagram showing cumulative output per unit over time with a linear trendline indicating diverted funds.
The Contracting Environment

- Agent reports cash flows
- Contract specifies, as function of the history of cash flows:
  - The agent’s compensation \( dC_t \geq 0 \)
  - Termination / Liquidation
    - Agent’s outside option = 0
    - Investors receive value of firm assets, \( L < \mu/r \)
- Contract curve / value function:
  \[ p(w) = \max \text{ investor payoff given agent’s payoff } w \]
  - Provide incentives via cash \( dC_t \) or promises \( dw_t \)
  - Tradeoff: Deferring compensation eases future IC constraints, but costly given the agent’s impatience
Solving the Basic Model

- First-Best Value Function
  \[ p^{FB}(w) = \frac{\mu}{r} - w \]

- Basic Properties
  - Positive payoff from stealing/shirking
    \[ \Rightarrow p(0) = L \]
  - Public randomization
    \[ \Rightarrow p(w) \text{ is weakly concave} \]
  - Liquidation is inefficient
    \[ \Rightarrow p(w) + w \leq \frac{\mu}{r} \]

- Cash Compensation
  \[ \Rightarrow p'(w) \geq -1 \]
  - Pay cash if \( w > w^c \)
  - Use promises if \( w \leq w^c \)
Basic Model cont’d

- Agent’s Future Payoff $w$
  - Promise-keeping
    - $E[dw] = \gamma \ w \ dt$
  - Incentive Compatibility
    - $\partial w / \partial y \geq \lambda$

  $\Rightarrow \ dw = \gamma \ w \ dt + \lambda \ (dy - E[dy])$

    $= \gamma \ w \ dt + \lambda \ \sigma \ dZ$

- Investor’s Payoff: HJB Equation

\[ rp = \mu + \gamma \ w p' + \frac{1}{2} \lambda^2 \sigma^2 p'' \]

Boundary Conditions:
- Termination: $p(0) = L$
- Smooth pasting: $p'(w^c) = -1$
- Super contact: $p''(w^c) = 0$

\[ p(w^c) + w^c = \frac{\mu - (\gamma - r) \ w^c}{r} \]

Agent’s Payoff $w$
The Gambling Problem

- Agent may increase profits by taking on tail risk
  - E.g. selling disaster insurance / CDS / deep OTM puts – earn $\rho \, dt$
  - Risk of disaster that wipes out franchise – arrival rate $\delta \, dt$, loss $D$
The Gambling Problem

• Agent’s incentives
  • Gain from gambling: $\lambda \rho \, dt$
  • Potential loss: $w_t$, with probability $\delta \, dt$
  • Agent will gamble if $\lambda \rho > \delta w_t$ or
    $$w_t < w^s \equiv \frac{\lambda \rho}{\delta}$$
  • Agent will gamble if not enough “skin in the game”

• Gambling region
  • Contract dynamics: $dw = (\gamma + \delta) w \, dt + \lambda (dy - E[dy])$
  • Value function: $(r + \delta) p^g = (\mu + \rho - \delta D) + (\gamma + \delta) wp^g' + \frac{1}{2} \lambda^2 \sigma^2 p^g''$
    • Increased impatience
  • Smooth pasting: $p(w^s) = p^g(w^s), \quad p'(w^s) = p^g'(w^s)$
Example

- **First Best = 100**
  - $\mu = 10$, $r = 10\%$, $\gamma = 12\%$, $\sigma = 8$, $L = 50$, $\lambda = 1$
- **Cash if $w > 56$**
  - $w^c = 56$
- **Gamble if $w < 40$**
  - $\rho = 2$, $\delta = 5\%$, $w^s = 40$, $D = 0$
- **Compare to pure cases**
  - Longer deferral of compensation
  - Greater use of credit line vs. debt (more financial slack)
Ex-Post Detection and Bonuses

• Suppose disaster states are observable
  • Earthquakes, Financial Crises, …
  • Can we avoid gambling by offering bonuses to survivors ex-post?

• How large a bonus?
  • If \( w_t \geq w^s \): no bonus is needed to provide incentives
  • If \( w_t < w^s \): increase \( w_t \) to \( w^s \) if firm survives disaster: \( b_t = w^s - w_t \)

• Bonus region
  • Contract dynamics: \( dw = [(\gamma + \delta) w - \delta w^s] dt + \lambda (dy - E[dy]) \)
  • Value function:
    \[
    (r + \delta) p^b = (\mu + \delta p^b(w^s)) + [(\gamma + \delta) w - \delta w^s] p^b' + \frac{1}{2} \lambda^2 \sigma^2 p^b''
    \]
  • Smooth pasting …
Optimal Bonuses

• Bonus payments:
  • substantially improve investor payoff
  • reduce need for deferred comp / financial slack / harsh penalties (no jumps)
• For low enough $w_t$, gambling is still optimal
Summary

• The double moral hazard problem is likely to be important in firms where risk-taking can be easily hidden
• Risk-taking is likely to take place
  • Probability of disaster is low
  • After a history of poor performance, when the agent has little “skin” left in the game
• As a result, optimal policies will have increased reliance on deferred compensation
• When the “safe” practices can be verified ex-post, we can mitigate risk-taking via bonuses
• When effort costs are convex, we should expect reductions in effort incentives as a means to limit risk-taking, with a jump to high powered incentives in the gambling region