Structural Breaks and Dynamic Characteristics of Inflation and Growth Rates of Monetary Aggregates

Igor Pelipas
BEROC

Minsk, September 2011
Belarusian Economic Research and Outreach Center is created in Kyiv as a joint project of the Stockholm Institute of Transition Economics, the Kyiv School of Economics, the Kyiv Economics Institute and the Economics Education and Research Consortium.

It is financed jointly by the Swedish International Development Cooperation Agency (SIDA) and by the United States Agency for International Development (USAID) through the Eurasia Foundation.
Abstract

The paper addresses the problem of determining the order of integration of inflation and growth rates of monetary aggregates under the multiple structural breaks in dynamics of these variables. Discussing the existent approaches for unit root and structural breaks testing, we propose the modified one where on the first stage the structural break points are determined endogenously by impulse indicator saturation technique and then the matching break points are utilized exogenously in the appropriate Dickey-Fuller unit root test. This approach allows unit root testing for any number of structural breaks. An application of the proposed approach to Belarusian data for 1995-2009 led us to conclusion that the rates of inflation both on the basis of GDP deflator and consumer price index, as well as the growth rates of monetary aggregates M0, M1, M, and M3 are the stationary variables with a changing mean. Consequently, these variables have the order of integration $I(0)$. The determined dates of structural breaks correspond to regime changes in the dynamics of the examined variables and have a clear economic interpretation. The results presented in the paper are useful for econometric modeling of inflation and monetary policy.

JEL Classification: C22, C51, E31, E51

Keywords: inflation, monetary aggregates, unit root test, structural brakes, impulse indicator saturation

* Belarusian Economic Research and Outreach Center, e-mail: pelipas@research,by
1. INTRODUCTION

The analysis of the dynamic properties of macroeconomic indicators is an important issue in economic literature. From the econometric perspective this issue is considered through determination the order of integration of analyzed variables using various tests for unit root or stationarity.\(^1\) If a nonstationary variable, usually represented in logs, becomes stationary in first differences, then the variable is said to be a unit root process and has the order of integration \(I(1)\). In turn, the first differences of a variable, that are stationary, have the order of integration \(I(0)\). For instance, if inflation rate is a stationary variable with the order of integration \(I(0)\), then the price level has a unit root and the order of integration \(I(1)\). On the contrary, if inflation rate is a nonstationary variable with the order of integration \(I(1)\), a price level has two unit roots and the order of integration \(I(2)\). All mentioned concerns the monetary aggregates as well since they have similar dynamics with the price indexes and often are considered in an interrelationship with prices.

The order of integration of price level and monetary aggregates stipulates the methodology of econometric modeling and forecasting with these variables. Additionally, the order of integration of price level is directly related to inflation persistence issue. If inflation is a stationary variable, i.e. persistence in the dynamics is absent or quite low, the various shocks will have only short term effects on inflation dynamics. After such shocks inflation will return to its steady states level. When, however, there is a unit root in inflation dynamics, inflation is a persistent process. In this case inflation has no tendency to return to its equilibrium level after a certain shock. Analysis of inflation persistence is important for carrying out of adequate monetary policy.

While testing the variables for unit root, often Dickey-Fuller (augmented) unit root tests are used (Dickey, Fuller (1979; 1981). These tests, as well known, have low power, and a probability of a type II error, when a false null hypothesis is accepted, is rather high. To avoid this problem, more powerful modified unit root test are proposed, where the initial data are

---

\(^1\) Pretesting of the modeling variables for unit root (stationarity) is a wide-spread exercise in time series econometrics. It should be noted, however, that some authors (see, for instance, Juselius (2006)) believe that stationarity (nonstationarity) of the macroeconomic variables is not an intrinsic feature of these variables. It is rather useful statistical approximation, permitting to classify short-run and long-run variation of the analyzed time series. For some (longer) periods the variable can be stationary and for other (shorter) periods it can be characterized as nonstationary. In the cointegrated vector autoregression framework it is possible to test for stationarity in a multivariate setting. In this paper the issues of unit root (stationarity) testing is considered in traditional univariate testing framework.
de-trended (de-meaned), using generalized least squares, and then these new data are utilized in the traditional Dickey-Fuller unit root test (Elliot, Rothenberg, and Stock (1996)). These and other unit root tests are widely presented in a various econometric packages.

Perron (1989), however, pointed out that structural break in time series makes the standard Dickey-Fuller unit root test biased towards non-rejection of false null hypothesis. In order to resolved this problem the various unit root tests are proposed that take into account the effect of one structural break (Perron (1990; 1992; 1997); Zivot, Andrews (1992)). In these tests such a break is determined exogenously or endogenously on the basis of analyzed data. The unit root tests that allow for two structural breaks are also put forward (Lumsdaine, Papell (1997); Lee, Strazicich (2003)).2 Usually, the unit root tests do not suppose more than two structural breaks and this point to some extend is a limitation of unit root testing. At the same time, there are testing procedures allowing to determine multiple structural breaks in the dynamics of variables (for instance, see Bai, Perron (1998; 2003)).

In this paper the dynamic properties of inflation rates (based on GDP deflator and consumer price index) and growth rates of various monetary aggregates (M0, M1, M2, and M3) in Belarus are analysed. As it was shown in Pelipas (2003) over the period 1992-2002 (monthly and quarterly data), the inflation rate based on CPI and the growth rates of monetary aggregates M0, M1, and M2 are stationary variables with changing means. Herewith, there was one clear structural break in 1995Q1-2. In current research the dynamics of the appropriate variables are considered for the period 1995-2009. Since during this period there were several structural breaks associated with external shocks and changes in monetary policy, we need to utilize relevant approaches permitted to take into account properly the influence of such events while testing for unit root.

The remainder of the paper is organised as follows. The second section provides description of the data used in the analysis. Some traditional methods of testing for unit root and structural breaks are also discussed in this section. The third section considers the method of impulse indicator saturation in testing for multiple structural breaks and proposes the modified approach for unit root testing under multiply structural breaks. On the basis of this approach the inflation rates based both on GDP deflator and consumer price index and growth rates of monetary aggregates M0, M1, M2, and M3 are tested for unit root. The last section concludes.

2 The unit root test, proposed in Lee, Strazicich (2003), assumes two structural breaks both for null and alternative hypothesis, while the unit root test in Lumsdaine, Papell (1997) does not presume the structural breaks for
2. DATA USED AND TRADITIONAL APPROACHES

The following indicators are tested for unit root (stationarity) in this research:\(^3\)

- GDP deflator index (DEFGDP);\(^4\)
- consumer price index (CPI);
- monetary aggregate M0 (cash in circulation);
- monetary aggregate M1 (M0 + transferable deposits in Belarusian rubles);
- monetary aggregate M2* (M1 + other deposits in Belarusian rubles + plus legal and/or natural persons’ funds in Belarusian rubles-denominated securities (except for shares) issued by the banks of the Republic of Belarus);
- monetary aggregate M3 (M2* + transferable deposits, other deposits in foreign currency, deposits in precious metals, and legal and/or natural persons’ funds in foreign currency-denominated securities (except for shares) issued by the banks of the Republic of Belarus.).

The analysis has been done for the period 1995-2009 on the basis of the quarterly data (60 quarters). The raw data are examined for seasonality and if it is detected, the appropriate seasonal adjustment of the data is carried out.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Model specification</th>
<th>The presence of seasonality (combined test for the presence of identifiable seasonality)</th>
<th>Quality assessment statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td>DEFGDP</td>
<td>((0, 1, 1) (0, 1, 1))</td>
<td>yes</td>
<td>0.44 &lt; 1</td>
</tr>
<tr>
<td>CPI</td>
<td>((0, 1, 1) (0, 1, 1))</td>
<td>yes</td>
<td>0.37 &lt; 1</td>
</tr>
<tr>
<td>M0</td>
<td>((2, 1, 2) (0, 1, 1))</td>
<td>yes</td>
<td>0.30 &lt; 1</td>
</tr>
<tr>
<td>M1</td>
<td>((0, 1, 1) (0, 1, 1))</td>
<td>yes</td>
<td>0.22 &lt; 1</td>
</tr>
<tr>
<td>M2*</td>
<td>((0, 1, 1) (0, 1, 1))</td>
<td>yes</td>
<td>0.37 &lt; 1</td>
</tr>
<tr>
<td>M3</td>
<td>((0, 1, 2) (0, 1, 1))</td>
<td>no</td>
<td>0.76 &lt; 1</td>
</tr>
</tbody>
</table>

In order to test for seasonality X–12 ARIMA model (Autoregressive Integrated Moving

null hypothesis that can lead to its false rejection.

\(^3\) All calculations in this paper are based on the data of the National Statistical Committee of the Republic of Belarus and the National Bank of the Republic of Belarus.

\(^4\) Calculations are made on the basis of GDP data in constant prices.
Average model) is utilized. Formally, ARIMA model can be described as \((p, d, q)(sp, sd, sq)\), where \(p\) is an autoregressive order of the model; \(d\) is an order of integration of the model; \(q\) is an order of moving average of the model; \(sp\), \(sd\), \(sq\) are the same notations, but for seasonal component of the model. Optimal specification of the ARIMA model is performed automatically. The final models are evaluated using \(Q\)-test (if the values of the test are not greater than one, it means that chosen specification of the model is statistically acceptable). The presence (absence) of seasonality in the analyzed time series is examined by means of the combined test for the presence of identifiable seasonality. Table 1 presents the results of testing for seasonality.

As it is evident from table 1, the specifications of ARIMA model chosen for all variables are statistically acceptable. The combined test for the presence of identifiable seasonality shows that seasonality presents in GDP deflator, consumer price index, monetary aggregates M0, M1, and M2. In accordance with this test, monetary aggregate M3 does not have statistically significant seasonal pattern. In the following analysis seasonal adjustment is made for those variables where it has been identified. Monetary aggregate M3 is not adjusted for seasonality.

Hereafter, all variables appear in natural logarithms (\(\ln\) is a natural logarithm, \(\text{SA}\) is an index, indicating that variable is seasonally adjusted) where \(\text{defgdp}_t = \ln\text{DEFGDP}_\text{SA} , \text{cpit}_t = \ln\text{CPI}_\text{SA} \), \(m_0_t = \ln\text{M0}_\text{SA} , m_1_t = \ln\text{M1}_\text{SA} , m_2_t = \ln\text{M2}_\text{SA}^* , m_3_t = \ln\text{M3}\) denote the logs of the levels of the variables; and \(\Delta\text{defgdp}_t = \text{defgdp}_t - \text{defgdp}_{t-1} , \Delta\text{cpit}_t = \text{cpit}_t - \text{cpit}_{t-1} , \Delta m_0_t = m_0_t - m_0_{t-1} , \Delta m_1_t = m_1_t - m_1_{t-1} , \Delta m_2_t = m_2_t - m_2_{t-1} , \Delta m_3_t = m_3_t - m_3_{t-1}\) are the first differences of the logs of the variables and the approximations of the growth rates. The log levels of all variables have upward trends and they are obviously nonstationary. Since we are interested in determination of the order of integration of the inflation rates and the growth rates of monetary aggregates, in the following analysis we consider only the first differences of the logs of the variables (figure 1).

The graphs of the variables presented in figure 1 do not give a clear picture concerning their order of integration. Dickey-Fuller unit root test (ADF) is frequently used in empirical research while formally determining the order of integration of the variables. As it was mentioned above, low power of this test can lead to the situation when false null hypothesis (the variable has a unit root) will not be rejected although, in fact, the variable is stationary. To handle this problem, one can apply more powerful modified Dickey-Fuller test (DFGLS),

---

5 X–12 ARIMA module in econometric package OxMetrics 6.2 is used for calculations.
which testing not actual variables but transformed, where the deterministic terms are removed (de-trending or de-meaning) by means of the generalized least squares (GLS). The results of these tests in different specifications are presented in table 2.

Figure 1. Dynamics of the inflation rate and the growth rates of the monetary aggregates

Table 2. Unit root tests without structural breaks

<table>
<thead>
<tr>
<th>Variable</th>
<th>Constant and trend</th>
<th>Constant</th>
<th>Without constant and trend</th>
<th>Constant and trend</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δdefgdp_t</td>
<td>-2.296(4)</td>
<td>-1.004(4)</td>
<td>-0.908(4)</td>
<td>-2.640(4)</td>
<td>-0.406(4)</td>
</tr>
<tr>
<td>Δcpi_t</td>
<td>-3.304(0)*</td>
<td>-3.090(0)**</td>
<td>-2.748(0)**</td>
<td>-2.704(0)</td>
<td>-1.331(0)</td>
</tr>
<tr>
<td>Δm0_t</td>
<td>-2.680(2)</td>
<td>-1.742(2)</td>
<td>-1.480(2)</td>
<td>-2.016(1)</td>
<td>0.028(2)</td>
</tr>
<tr>
<td>Δm1_t</td>
<td>-4.782(0)****</td>
<td>-4.141(0)****</td>
<td>-3.072(0)**</td>
<td>-2.694(0)</td>
<td>-1.004(0)</td>
</tr>
<tr>
<td>Δm2_t</td>
<td>-5.311(0)****</td>
<td>-4.500(0)****</td>
<td>-3.023(0)**</td>
<td>-3.104(0)*</td>
<td>-1.272(0)</td>
</tr>
<tr>
<td>Δm3_t</td>
<td>-4.165(0)****</td>
<td>-1.136(4)</td>
<td>-1.057(4)</td>
<td>-3.989(0)**</td>
<td>-0.667(4)</td>
</tr>
</tbody>
</table>

Notes: *, ** and *** denote rejection of null hypothesis at the ten per cent, five per cent and one per cent significance level respectively. t-ADF(n) and t-DFGLS(n) is t-statistic in ADF and DFGLS unit root tests; n is an optimal lag length, chosen by modified Akaike information criteria. Maximal lag length is 4 quarters. Calculations are made in econometric package Eviews 7.2.
The obtained results do not provide unambiguous answer to the question: are analyzed time series stationary or unit root process. According to unit root tests presented above, the variables $\Delta \text{defgdp}$, and $\Delta m0$, are not stationary. As for the other variables, unit root null hypothesis is rejected by ADF-test and it is not rejected while using DFGLS-test ($\Delta m3$, in specification with constant and trend is an exception). It is important to note that the results of ADF-test in our case are very sensitive to sample period and lag length. The results presented in table 2, are obtained for full sample and optimal lag length chosen by the modified Akaike information criteria. However, if the lag length would be chosen so to remove autocorrelation of the residuals and at the same time the sample would be shifted barely two quarters ahead, then the unit root null hypothesis will not be rejected for any examined variables. Hence, the usage of traditional tests for unit root without structural breaks in our case does not provide the reliable and noncontradictory results.

Figure 1, besides graphs of the variables, presents the means of the variables for the total sample. These means, obviously, do not characterize the peculiarities of the dynamics of inflation rates and the growth rates of monetary aggregates. During the whole period there were structural breaks due to economic policy measures and external shocks. The presence of such structural breaks and specific breaking points could be determined econometrically using the approach proposed in Bai and Perron (1998, 2003).

Within this approach the sum of squared residuals is minimized in order to identify the dates of $k$ structural breaks in time series $\Delta y_t$ and, thereby, determine $k + 1$ regime in dynamics of examined variable, on the basis of the following model:

$$\Delta y_t = \gamma_{k+1} + \tau_t,$$

where $\Delta y_t$ is variable; $\gamma_{k+1}$ is a series of $k + 1$ constants that characterized the means of the variable in each of $k + 1$ regimes; $\tau_t$ is regression residuals.

The model (1) is corrected for autocorrelation by reservation of a certain share of the sample corresponding to minimal regime duration (usually 0.15 from total sample). The final model is chosen using Bayesian information criterion.

The dates of the structural breaks, specified on the basis of the model (1), are presented in table 3. As it is evident from the results, there are structural breaks (changing means) in the dynamics of variables. For all variables, but $\Delta m0$, Bai-Perron test denotes several such changes. These structural breaks evidently should be taken into consideration in unit root testing to get adequate results. It should be pointed out that Bai-Perron technique for determining
structural breaks presupposes the stationarity of the testing variables. Thus, it is not the best choice in determination of breaking points while testing for unit root.

Table 3. Structural breaks in the dynamics of inflation rates and the growth rates of monetary aggregates

<table>
<thead>
<tr>
<th>Variable</th>
<th>The dates of the structural breaks (changing mean), year and quarter</th>
<th>Number of different regimes</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta \text{defgdp}_t$</td>
<td>1998:3; 2000:4</td>
<td>3</td>
</tr>
<tr>
<td>$\Delta \text{cpi}_t$</td>
<td>1998:3; 2000:3; 2003:3</td>
<td>4</td>
</tr>
<tr>
<td>$\Delta m_0$</td>
<td>2001:3</td>
<td>2</td>
</tr>
<tr>
<td>$\Delta m_1$</td>
<td>2000:3; 2006:4</td>
<td>3</td>
</tr>
<tr>
<td>$\Delta m_2$</td>
<td>2004:4; 2006:3</td>
<td>3</td>
</tr>
<tr>
<td>$\Delta m_3$</td>
<td>1998:3; 2000:4</td>
<td>3</td>
</tr>
</tbody>
</table>

Notes: trimming factor is equal 0.15 that corresponds to 9 quarters for our sample (60 quarters). The calculations are carried out having used add-in procedure “Bai-Perron breakpoint test” in econometric software Eviews 7.2 and R package.

Despite the above remarks, it is reasonable to suppose that there are structural breaks in the dynamics of the examined variables and to apply the appropriate tests for unit root which permit to take these structural breaks into account. For this purpose we apply minimum LM-unit root test, proposed in Lee and Strazicich (2003). In this unit root test structural break points are determined endogenously. In principle, it is possible to set any number of structural breaks in this unit root test. However, in fact the test is elaborated only for at most two structural breaks with appropriate critical values.

In general, the results, presented in table 4, do not give evidence in favor of stationarity of the analyzed variables under structural breaks in means ($\Delta m_3$ is an exception). Moreover, the dates of the structural breaks, endogenously determined within the minimum LM unit root test, in some instances are substantially different from those obtained by Bai-Perron breakpoint test. The coefficients at the dummies, characterizing structural breaks, in the most cases are statistically insignificant. Additionally, the different lag structure in the minimum LM unit root test significantly affect the choice of the structural break points but ultimately this has no effect on the results of the test themselves. Thereby, one can conclude that the tests considered above, both without and with structural breaks, in our case do not provide clear cut answer on the main research question of the paper, namely, are inflation rate and the growth rates of monetary aggregates in Belarus stationary variables or not. Moreover, the unit root tests utilized above are very sensitive even to small changes of sample. In its turn, Bai-Perron break points test initially supposes stationarity of the variables and needs to be corrected for
serial correlation by trimming the initial sample.

### Table 4. Minimum LM unit root test with structural breaks

<table>
<thead>
<tr>
<th>Variable</th>
<th>$t$-LM$_t(n)$</th>
<th>The dates of structural breaks ($t$-statistics)</th>
<th>Critical values (5%; 1%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta defgdpt$</td>
<td>-3.284(0)</td>
<td>1998:4 (2.764) 2006:1 (0.514) –</td>
<td>-3.842; -4.545</td>
</tr>
<tr>
<td>$\Delta cpi_t$</td>
<td>-3.006(0)</td>
<td>1998:1 (1.134) 2000:3 (-0.914) 2005:1(0.225)</td>
<td>-3.842; -4.545</td>
</tr>
<tr>
<td>$\Delta m0_t$</td>
<td>-2.378(0)</td>
<td>2003:2 (1.062) – –</td>
<td>-3.842; -4.545</td>
</tr>
<tr>
<td>$\Delta m1_t$</td>
<td>-2.766(0)</td>
<td>1998:3 (2.823) 2003:2 (0.883) –</td>
<td>-3.842; -4.545</td>
</tr>
<tr>
<td>$\Delta m2_t$</td>
<td>-3.388(0)</td>
<td>1998:3 (1.606) 2002:3 (0.322) –</td>
<td>-3.842; -4.545</td>
</tr>
<tr>
<td>$\Delta m3_t$</td>
<td>-4.701(0)**</td>
<td>1998:4 (4.084) 2000:4 (-1.960) –</td>
<td>-3.842; -4.545</td>
</tr>
</tbody>
</table>

**Notes:** ** *** denote rejection of null hypothesis at the one per cent significance level. $t$-LM$_t(n)$ is $t$-statistic in minimum LM unit root test; $n$ is optimal lag length chosen by “general-to-specific” approach. The Numbers in parentheses at the dates of structural breaks are $t$-statistic for appropriate dummies. Critical values are taken from Lee and Strazicich (2003) for the model with changing mean (crash model). As far as the critical values for three structural breaks are not available, the values for two breaks are presented in the table. The computations are conducted using econometric package Rats 7.1 and procedure “lsunit.src”.

#### 3. UNIT ROOT OR STATIONARITY: ANOTHER APPROACH

In this section we propose a modified approach for unit root testing of the variables with changing mean. The essence of this approach is as follows:

First, the break point in the dynamics of the analyzed variables are determined endogenously using multiple structural breaks test based on impulse indicator saturation (IIS) method (Santos (2008), Castle, Doornik, and Hendry (2010)).

Second, on the basis of the impulse indicator saturation break test mentioned above, step dummies are created; these step dummies characterize different regimes in dynamics of the analyzed variables and reflect changes in a variable mean.

Third, the step dummies created on the previous stage are included in Dickey-Fuller unit root test by analogy with dummy variables included in cointegrated vector in the Johansen (1988) multivariate cointegration test.

Fourth, testing for null hypothesis of unit root, $t$-statistics in Dickey-Fuller test ($t$-ADF) are compared with appropriate critical values calculated for cointegration test in the conditional equilibrium correction model framework (Ericsson and MacKinnon (2002)). At that,
the dummy variables included in Dickey-Fuller unit root test are considered as additional variables in cointegration test. Let us discuss this approach and its basics in more detail.

3.1. Structural breaks and impulse indicator saturation

Method of impulse indicator saturation is one of the latest developments in econometric modeling (Hendry, Johansen, and Santos (2008); Johansen and Nielsen (2009); Hendry and Santos (2010)). To analyse the properties of econometric model, this method uses zero-one impulse indicator dummies. Since there are potentially \( T \) such dummy variables, inclusion all of them in a model is infeasible. However, impulse indicator dummies can be included in a model as the separate blocks. In the simplest case with two blocks the sample is split on two equal parts \( (T/2) \), then impulse indicator dummies are included only for the first half of the sample and statistically significant dummies at a chosen significant level \( \alpha \) are stored. Further, chosen at the previous step impulse indicator dummies are dropped, and then another part of the dummies are included in the model. Then procedure is repeated for the second part of the sample. Statistically significant impulse indicator dummies from two blocks are combined and jointly significant are retained.

A computational algorithm, utilized in econometric package OxMetrics 6.2, performs optimal splitting for any number of blocks selecting the final model. Method of impulse indicator saturation allows to determine the structural breaks, outliers and possible data contaminations in econometric modeling. Applications of this method for analysis of structural breaks (changing mean) while studying inflation persistence are presented in Santos and Oliveira (2010), Oliveira and Santos (2010).

3.2. Impulse indicator saturation breaks test: a hypothetical example

Let us consider an example of impulse indicator saturation breaks test using Monte Carlo simulation. The parameters of Monte Carlo experiment have been chosen quite arbitrarily, although we were trying to catch the peculiarities of real dynamics of the variables analyzed in this paper. Suppose that there is a structural break (change of the mean) in the dynamics of the variable \( y_t \) inside the sample (such conditions in fact mimic two structural breaks, that to a some extend brings in line with actual dynamics of inflation rates and growth rates of monetary aggregates during the investigated period). The points of these structural breaks are known: \( T_{1b} = 30 \) and \( T_{2b} = 45 \). At that, regimes changes in the dynamics of the variable are happened in the \( T_{1b} + 1 \) and \( T_{2b} + 1 \). Total sample is equal to 100 observations. Suppose also that the data generating process for variable \( y_t \) is represented by the following autoregression:
\[ y_t = \mu + \varphi y_{t-1} + \varepsilon_t, \quad \varepsilon \sim N(0,1). \]  

Herewith, an autoregression parameter is \( \varphi = 0.5 \) (stationary process) in all regimes. A constant is \( \mu = -0.5 \) before and after the structural break, and \( \mu_b = 1.5 \) for the segment of structural break. Residuals \( \varepsilon_t \) in (2) are normally independently distributed with zero mean and unit variance. Therefore, we specify a stationary process with changing mean inside the sample that supposes three regimes in the dynamics of the variable \( y_t \). Monte Carlo experiment assumes the number of replications \( M = 1000 \). The resulting experiment variable is represented on the top graph in figure 2.

![Figure 2. Structural breaks and impulse indicator saturation](image)

Thereafter we employ impulse indicator saturation to detect the structural breaks and its points in dynamics of the variable \( y_t \), including only a constant as a regressor in the following model:

\[ y_t = \mu + \varepsilon_t. \]  

6 The computation were made using module PcNaive in econometric package OxMetrics 6.2.
A significance level $\alpha = 0.01$ is used when estimating the model by impulse indicator saturation. The obtained results are visualized in the mid graph in figure 2. The line denoted as IIS shows disposition of statistically significant indicator variables in the model (the segments depicted as a straight line correspond to insignificant indicator variables). In this case the structural breaks are identified as continuous sequence of statistically significant indicator variables with the same signs and approximately the same magnitudes in the model (3). Several statistically significant indicator variables represent outliers. As one can see, the structural break in figure 2 is determined as the segment between 30 and 45 observations.\footnote{The results are obtained using \textit{Autometrics} routine, selecting the model by the “general-to-specific” approach with impulse indicator saturation in econometric package OxMetrics 6.2.}

Analysis of the coefficients of impulse indicator variables in the model (3) shows that this test specifies the presence of structural break in the dynamics of the variable $y_t$ and exactly identifies the shifting points in the mean of the variable initially set in our Monte Carlo experiment, i.e. thirtieth and forty-fifth observation (regime in the dynamics of the variable is changed from 31 and from 46 observation, i.e. in the points $T_{1b} + 1$ and $T_{2b} + 1$). In this segment the coefficients have the same sign and comparable magnitudes: the mean of the coefficients is equal to 5.526, with maximum and minimum values equal 6.243 and 4.264 respectively. To take into account the mean shift identified within the impulse indicator saturation break test, the following two step dummies are created: $D_{1t} = 1(t \geq T_{1b} + 1)$ and $D_{1t} = 0(t \leq T_{1b})$; $D_{2t} = 1(t \geq T_{2b} + 1)$ and $D_{2t} = 0(t \leq T_{2b})$. An appropriate regression reflecting changes in mean for variable $y_t$ is depicted on the bottom graph in figure 2. It is interesting to note that Bai-Perron break point test discussed earlier in the paper, identified three break points in our hypothetical example, namely 30th and 46th observation (that is in principle in accordance with data properties) and 78th observation (that is not in line with the actual data generation process).

### 3.3. Determination of structural breaks using impulse indicator saturation: actual data

In this section we applied impulse indicator saturation break (mean shift) test discussed above to identify breaks in dynamics of inflation rates and growth rates of monetary aggregates in Belarus. For visualization and to save space the obtained results are presented only in graphical form (figure 3). The estimations are performed on the basis of the model (3) and significance level $\alpha = 0.025$ is used. In this case we consider the structural break as continuous sequence of statistically significant indicator variables with the same signs and approximately
the same magnitudes (continues sequence equal to 6 quarters are chosen arbitrary for practical reasons). Other statistically significant indicator variables are treated as outliers. The third structural breaks in the dynamics of the variables \( \Delta m_1 \) and \( \Delta m_2 \), is an exception, where the regime is determined with the omission of three quarters due to the influence of global economic and financial crisis.

Figure 3. Structural breaks in dynamics of inflation rates and growth rate of monetary aggregates

Index lines in figure 3 graphically represent the results of structural break test. Continuous sequences of statistically significant indicator variables form the segments that characterized the changes of regimes in the dynamics of the examined variables. On this basis the step dummies that take into account the changes in means of the variables are created (dotted line in figure 3). It is evident that all variables in accordance with this test have the structural breaks in their dynamics. There are two structural breaks in the dynamics of \( \Delta defgdp_i \), \( \Delta cpi_i \), \( \Delta cpi_i \) and \( \Delta cpi_i \) (3 regimes), whereas the dynamics of \( \Delta m_1 \) and \( \Delta m_2 \) characterized by three structural breaks (4 regimes). The specific dates of the structural breaks that have been obtained by impulse indicator saturation break test are presented in table 5.

These results are different from those that were presented earlier in table 3. Additionally, the results of structural break test based on impulse indicator saturation are clearly con-
sistent with real dynamics of the variables and the break points have explicit economic interpretation. Specifically, the structural break in 1998 Q1-2 is caused by the Russian financial crisis in August 1998. The structural break in 2000 Q2-4 and 2001 Q1 (for different variables) occurs due to adoption of unified exchange rate for Belarusian ruble and the following changes of monetary policy. Finally, the structural break in the beginning of 2007 is related to tightening of monetary policy in order to eliminate the impact of energy price growth on exchange market. Since all break points have a clear-cut economic interpretation, the inclusion of the appropriate dummies, taking into account the impact of such breaks in a unit root test, is not simple “fitting” of the regression; it is based on a solid economic ground. It is also important that the break points are chosen endogenously within impulse indicator saturation break test and they reflect real peculiarities of the examined times series.

### Table 5. Structural breaks in the dynamics of inflation rates and the growth rates of monetary aggregates: impulse indicator saturation break test

<table>
<thead>
<tr>
<th>Variable</th>
<th>The dates of the structural breaks (changing mean), year and quarter</th>
<th>Number of different regimes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Δdefgdp,</td>
<td>1998:2; 2000:4</td>
<td>3</td>
</tr>
<tr>
<td>Δcpi,</td>
<td>1998:3; 2000:2</td>
<td>3</td>
</tr>
<tr>
<td>Δm0,</td>
<td>1998:3; 2001:1</td>
<td>3</td>
</tr>
<tr>
<td>Δm1,</td>
<td>1998:3; 2000:3; 2006:4</td>
<td>4</td>
</tr>
<tr>
<td>Δm2,</td>
<td>1998:3; 2000:3; 2006:4</td>
<td>4</td>
</tr>
<tr>
<td>Δm3,</td>
<td>1998:2; 2000:4</td>
<td>3</td>
</tr>
</tbody>
</table>

### 3.4. Unit root test with structural breaks determined by impulse indicator saturation

The break points specified endogenously by impulse indicator saturation break test can be utilized as exogenous in Dickey-Fuller unit root test using the appropriate step dummies. Such step dummies take a values $D_t = 1(t \geq T_b + 1)$ and $D_t = 0(t \leq T_b)$, i.e. they are equal to zero up to and including structural break point and equal to unity after the point of structural break. Inclusion of such dummies in Dickey-Fuller unit root test in principle is not a problem and this issue is discussed, for instance, in Perron (1990; 1992). However, usually one (sometimes two) dummy variable is included in Dickey-Fuller unit root test and available critical values (both for exogenously and endogenously determined break point) do not permit to test for unit root with more number of structural breaks. We believe that this issue can be handled by analogy with multivariate cointegration test with structural breaks (Johansen, Mosconi, and
Consider the following standard Dickey-Fuller unit root test in the specification assuming constant as a deterministic term:

\[ \Delta y_t = \mu + \alpha y_{t-1} + \sum_{i=1}^{k} c_i \Delta y_{t-i} + \epsilon_t, \]  

where \( \Delta y_t = y_t - y_{t-1} \); \( \mu, \alpha, c_i \) are parameters of the regression; \( \epsilon_t \) is an error term.

Dickey-Fuller unit root test intrinsically is a univariate case of the vector autoregression model with equilibrium correction mechanism. If the variable in (4) is stationary, then this variable is cointegrated with itself and a coefficient \( \alpha \) in (4) will have the following property: \(-1 \leq \alpha < 0\). The result will be an equilibrium correction of the variable when it departs from equilibrium level after various shocks. In this case the coefficient \( \alpha \) in the regression (4) is similar to so-called feedback coefficients in Johansen multivariate cointegration test characterizing the speed of the equilibrium correction of the system. In this context it is possible to reformulate Dickey-Fuller unit root test treating the multiple changes of the mean as in vector autoregression model with equilibrium correction mechanism in the case when constant are included in cointegration space:

\[ \Delta y_t = [\mu + \alpha y_{t-1} + \sum_{i=1}^{n} \phi_i D_{i,t-1}] + \sum_{i=1}^{n} \sum_{j=0}^{m-1} \beta_{ij} \Delta D_{i,j,t-j} + \sum_{j=1}^{m-1} \gamma_j \Delta y_{t-j} + \epsilon_t, \]  

where \( D_{i,t} = 1(t \geq T_{bi} + 1) \); \( T_{bi} \) is point of the \( i \)-th structural break; \( \Delta D_{i,t} = D_{i,t} - D_{i,t-1} \), \( \mu, \alpha, \phi_i, \beta_{ij}, \gamma_j \) are parameters of the regression; \( \epsilon_t \) is an error term; \( n \) is a number of step dummies characterizing changes in the mean of variable; \( m \) is a number of lags in the regression.

In the brackets in the regression (5) “the long-term” component of the variable’ dynamics is marked. This component is composed of the constant, characterizing the mean of the variable, the step dummies, reflecting the changes in the mean and taken as in Johansen cointegration procedure with one lag, and the variable itself with one lag. By analogy with Johansen cointegration test “short-term” part of the regression (5) includes the lags of dependent variable and lags of the first difference of step dummies. Thus, we have the equilibrium correction model but for only one variable with the set of deterministic terms (constant and step dummies). The coefficient \( \alpha \) in the regression (5) one can treat as an equilibrium correction mechanism and its significance in (5) can be tested using critical values from the cointegration test for conditional equilibrium correction model elaborated in Ericsson and MacKinnon (2002). At that, dummies in regression (5) can be considered as the additional variables in cointegration vector and then we can use critical values in accordance with the total number of such variables. If the break points are preliminary determined endogenously using impulse
indicator saturation technique, then proposed approach permits unit root testing practically with any number of structural breaks.

Table 6 reports the results of unit root tests of the analysed variables. The lag structure of the model is chosen so to eliminate residual autocorrelation in (5). For all variables the specification with zero lag was sufficient to eliminate autocorrelation. The coefficients at the dummies, characterizing changes in the mean, are statistically significant. Their signs correctly indicate the directions of the regimes changes in dynamics of the variables. As it was mentioned earlier, all structural breaks have clear-cut economic interpretation. According to t-ADF the null unit root hypothesis is rejected at 1 per cent significance level for all variables. Therefore, inflation rates based on GDP deflator and consumer price index ($\Delta\text{defgdpt}$ and $\Delta\text{cpit}$) as well as growth rates of the monetary aggregates ($\Delta m_0$, $\Delta m_0$, $\Delta m_2$, and $\Delta m_3$) are stationary variables with changing means and respectively they have an order of integration $I(0)$. Price levels and levels of monetary aggregates, in turn, have the order of integration $I(1)$. These results have an implication for appropriate choice of econometric methodology when modelling and forecasting inflation and for monetary policy as well.

### Table 6. Unit root test with multiply structural breaks

<table>
<thead>
<tr>
<th>Variable</th>
<th>t-ADF(n)</th>
<th>Dates of structural break</th>
<th>AR 1-4 (p-value)</th>
<th>Critical values (5%; 1%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta\text{defgdpt}$</td>
<td>$-6.10(0)^{**}$</td>
<td>1998:2 2000:4 –</td>
<td>0.2948</td>
<td>$-4.03; -4.76$</td>
</tr>
<tr>
<td></td>
<td>(4.76)</td>
<td>(−5.33)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta\text{cpit}$</td>
<td>$-7.52(0)^{**}$</td>
<td>1998:3 2000:2 –</td>
<td>0.3006</td>
<td>$-4.03; -4.76$</td>
</tr>
<tr>
<td></td>
<td>(4.02)</td>
<td>(−4.65)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta m_0$</td>
<td>$-7.40(0)^{**}$</td>
<td>1998:3 2001:1 –</td>
<td>0.5952</td>
<td>$-4.03; -4.76$</td>
</tr>
<tr>
<td></td>
<td>(3.05)</td>
<td>(−4.22)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta m_0$</td>
<td>$-6.35(0)^{**}$</td>
<td>1998:3 2000:3 2006:4</td>
<td>0.1713</td>
<td>$-4.40; -5.16$</td>
</tr>
<tr>
<td></td>
<td>(2.61)</td>
<td>(−3.352) (−2.64)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta m_2$</td>
<td>$-6.58 (0)^{**}$</td>
<td>1998:3 2000:3 2006:4</td>
<td>0.0607</td>
<td>$-4.40; -5.16$</td>
</tr>
<tr>
<td></td>
<td>(2.80)</td>
<td>(−3.48) (−2.48)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\Delta m_3$</td>
<td>$-6.45(0)^{**}$</td>
<td>1998:3 2000:4 –</td>
<td>0.5704</td>
<td>$-4.03; -4.76$</td>
</tr>
<tr>
<td></td>
<td>(3.99)</td>
<td>(−4.95)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes**: ** denote rejection of null hypothesis at the one per cent significance level. t-ADF(n) is t-statistic in ADF-test for unit root with changing mean; n is the lag length chosen so to eliminate residual autocorrelation. In the parentheses below the date of the structural break t-statistics at the appropriate dummies are presented. AR 1-4 is F-test for serial correlation of residuals of 1-n-order, $H_0$: serial correlation is not present. Critical values are determined on the basis of Ericsson and MacKinnon (2002). Computations are carried out using econometric package OxMetrics 6.2 (Doornik, Hendry (2009)).
4. CONCLUSION

The paper analysed the dynamic properties of the quarterly inflation rates based both on GDP deflator and consumer price index and the growth rates of monetary aggregates M0, M1, M1, and M3 in Belarus over the period 1995-2009, proposing the novel approach for unit root testing in the presence of multiply structural breaks (changing mean). Utilizing a newly developed impulse indicator saturation technique, the structural breaks and associated regimes in the dynamics of these variables have been specified endogenously. All specified structural breaks have the clear-cut economic interpretation, namely, the structural break in 1998 Q1-2 is caused by the Russian financial crisis in August 1998; the structural break in 2000 Q2-4 and 2001 Q1 occurs due to adoption of unify exchange rate for Belarusian ruble and the following changes of monetary policy; the structural break in the beginning of 2007 is related to tightening of monetary policy in order to eliminate the impact of energy price growth on exchange market.

The results of a unit root test with specified structural breaks demonstrated that a null unit root hypothesis is rejected for all examined variables. Therefore, the rates of inflation both on the basis of GDP deflator and consumer price index, as well as the growth rates of monetary aggregates M0, M1, M, and M3 are the stationary variables with a changing mean and have the order of integration \( I(0) \). In its turn, the levels of these variables are nonstationary variables with the order of integration \( I(1) \). The results obtained in the paper matter for econometric modelling of examined variables (for example, the usage of cointegration methodology, models with equilibrium correction mechanism, etc.) and monetary policy (stationarity of inflation rate testifies that it is not a persistent variable and after various shocks it will return to equilibrium level).

The approach for unit root testing proposed in the paper can be utilized for any variables where the structural breaks in the form of changing means are suspected. The important feature of this approach is the possibility to take into account practically any number of structural breaks, while traditional approaches usually permit to include into test only one or two structural breaks.

REFERENCES


Bai, J., Perron, P. 2003. Computation and Analysis of Multiple Structural Change Models,


